Answers to exam-style questions

Option A

1 ✓ = 1 mark

1 a A reference frame is a set of rulers and clocks at every point in space. ✓
used to record the position and time of events. ✓

b The Maxwell theory predicts that the speed of light is universal constant. ✓
Independent of the speed of the source. ✓

c i This observer measures a non-zero magnetic field due to the current. ✓
And hence a magnetic force on the moving charged particle. ✓

ii The observer moving along with the particle will measure an electric force on the particle. ✓
Due to the fact that the wire appears to be electrically charged. ✓

2 a The length of an object as measured in the rest frame of the object. ✓

b $\gamma = \frac{1}{\sqrt{1-0.98^2}} = 5.0$ ✓

Hence the measured length is $\frac{480}{5.0} = 96$ m. ✓

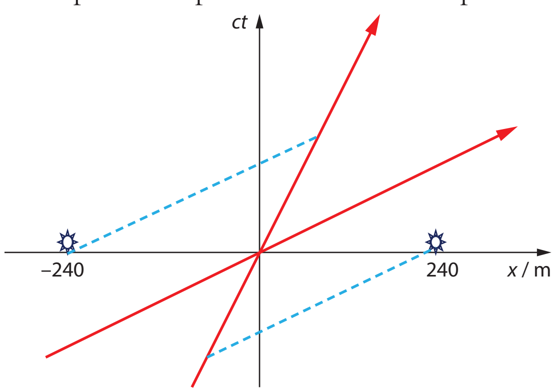
c i Light from each of the lamps will reach the observer at rest at the origin of the space station frame at the same time according to all observers. ✓
The spacecraft observers see the space station observer to move away from the light from the right lamp and towards the light from the left lamp. ✓
Light from the two lamps is moving towards the space craft observer at the same speed *c*. ✓
Hence for the light to arrive at the same time the light from the right must have been emitted first. ✓

ii $\Delta x = 480$ m and $\Delta t = 0$ ✓

Hence for the space craft observers $\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) = 5.0 \times \left(0 - \frac{0.98c \times 480}{c^2}\right) = -7.8 \times 10^{-6}$ s . ✓

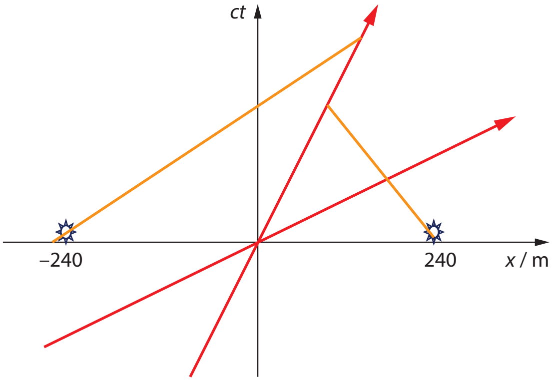
$\Delta t' = -7.8 \times 10^{-6}$ s ✓

d Lines parallel to primes *x* axis from lamps as shown below. ✓



e i Line at −45°. ✓
Intersecting the primed *t* axis. ✓

ii Line at 45° ✓
Intersecting the primed t axis. ✓



3 a i $u = \dfrac{0.78c + 0.50c}{1 + 0.78 \times 0.50}$ ✓

$u = +0.92c$ ✓

ii $u = \dfrac{0.78c + (-0.50c)}{1 + 0.78 \times (-0.50)}$ ✓

$u = +0.46c$ ✓

iii $u' = \dfrac{v - u}{1 - \dfrac{vu}{c^2}} = \dfrac{0.50c - (-0.50c)}{1 - 0.50 \times (-0.50)}$ ✓

$u = +0.80c$ ✓

b $u = \dfrac{v + c}{1 + \dfrac{v}{c}}$ ✓

$u = c$ ✓

4 a The twin paradox refers to a situation where two twins which are initially at the same place are separated when one of the twins flies away in a rocket and returns some time later. The "paradox" lies in that the Earth bound twin claims she is at rest and so her brother is younger when he returns. ✓
Whereas the brother may claim that in reality it is he who is at rest and he is older when the twins are reunited. ✓

b The "paradox" is resolved by noticing that the Earth bound twin is always in the same inertial frame. ✓
Whereas the traveller changes frames and so is younger. ✓

c i This is the time according to Earth clocks when the rocket gets to the planet, i.e. the reading is $\dfrac{12\text{ ly}}{0.80c} = 15$ year. ✓

ii This is the reading of the rocket clock when the rocket gets to the planet so it is the proper time interval for the duration of the trip. ✓
Hence the reading is $\dfrac{15}{\gamma} = \dfrac{15}{5/3} = 9.0$ year. ✓

iii The rocket clock reading of 9.0 yr is the time dilated interval of the reading of the Earth clock at A. ✓
And so the reading we seek is $\dfrac{9.0}{\gamma} = \dfrac{9.0}{5/3} = 5.4$ year. ✓

iv As in **iii** the incoming rocket time interval from T to R of 9.0 years is the time dilated interval from C to R $\dfrac{15}{\gamma} = \dfrac{15}{5/3} = 9.0$ year. ✓
Hence the interval from C to R is 5.4 years and so the reading at C is $30 - 5.4 = 24.6$ year. ✓

v The trip took 15 year out and another 15 yr in, so the reading at R for the earth clock is 30 year. ✓

vi For the rocket, the trip took 9 year out and another 9 yr in, so the reading at R for the incoming rocket's clock is 18 year. ✓

d Using the previous answers the Earth bound twin aged by 30 year. ✓
And the rocket twin by 18 year. ✓

5 a i $\frac{690}{0.75c} = 3.1\times 10^{-6}$ s ✓

ii For the space craft observers $\Delta t' = 3.1\times 10^{-6}$ s and $\Delta x' = 690$ m. ✓
The gamma factor is 1.90. ✓

Hence $\Delta t = \gamma\left(\Delta t' + \frac{v\Delta x'}{c^2}\right) = 1.90\times\left(3.1\times 10^{-6} + \frac{0.85c\times 690}{c^2}\right) = 9.6\times 10^{-6}$ s. ✓

b None is proper. ✓
Because the two events happen at different points in space. ✓

c $u = \frac{0.85c + 0.75c}{1 + 0.85\times 0.75}$ ✓
$u = +0.98c$ ✓

d i $x = ut = 0.98c \times 9.6\times 10^{-6}$ ✓
$x = 2.8\times 10^{3}$ m ✓

ii For the space craft observers $\Delta t' = 3.1\times 10^{-6}$ s and $\Delta x' = 690$ m ✓
Hence $\Delta x = \gamma(\Delta x' + v\Delta t') = 1.90\times(690 + 0.85\times 3\times 10^{8}\times 3.1\times 10^{-6}) = 2.8\times 10^{3}$ m. ✓

6 a i $\frac{6.0\text{ ly}}{0.60c} = 10$ year ✓

ii The gamma factor is 1.25. ✓
Hence the time is $\frac{10}{1.25} = 8.0$ year. ✓

b i The signal moves at the speed of light. ✓
$\frac{6.0\text{ ly}}{c} = 6.0$ year ✓

ii According to the spacecraft the distance separating the earth and the craft at the time of emission of the signal is $\frac{6.0}{1.25} = 4.8$ ly. ✓
In the time T it takes the signal to get to earth the Earth moves away distance 0.60cT. ✓
Hence $cT = 4.8 + 0.60cT$. ✓
Giving $T = \frac{4.8\text{ ly}}{0.40c} = 12$ year. ✓

7 i For an observer on the ground, without time dilation the muons would travel a distance of $0.98c\times 2.2\times 10^{-6} = 646.8 \approx 650$ m and then decay into electrons. ✓
With time dilation they would travel a distance of $0.98c\times\gamma\times 2.2\times 10^{-6} = 5.0\times 646.8 = 3234 \approx 3200$ m and then decay into electrons. ✓
The experimental fact that muons, rather than electrons, are detected at the Earth's surface is evidence for time dilation. ✓

ii For an observer in the muon's rest frame the Earth is moving upwards and would cover a distance of $0.98c\times 2.2\times 10^{-6} = 646.8 \approx 650$ m as the muons decayed into electrons. ✓
With length contraction the distance separating the surface form this observer is $\frac{3000}{\gamma} = \frac{3000}{5.0} \approx 600$ m. ✓
Hence when the particles arrive at the surface they are muons. ✓

8 a i The kinetic energy gained is the work done which is 2.5 GeV. ✓
Hence the total energy is 2.5 + 0.938 = 3.4 GeV. ✓

ii From $E^2 = p^2c^2 + (mc^2)^2$ we find $pc = \sqrt{E^2 - (mc^2)^2} = \sqrt{3.438^2 - 0.938^2} = 3.3\ \text{GeV}$. ✓
Hence $p = 3.3\ \text{GeV c}^{-1}$. ✓

iii $E = \gamma mc^2 \Rightarrow 3.4 = \gamma \times 0.938$ so that $\gamma = 3.6$. ✓
From $p = \gamma mv$ we find $3.3\ \text{GeV c}^{-2} = 3.6 \times 0.938\ \text{GeV c}^{-2} \times v$. ✓
Hence $v = \frac{3.3}{3.6 \times 0.938}\ \text{c} = 0.98\,\text{c}$. ✓

b The gamma factor for 0.98 c is 5.0. ✓
The total energy of one of the incoming particles is therefore 675 MeV, and the total energy is 1350 MeV. ✓
The particle is produced at rest (the initial momentum is zero) and so its rest mass is $1350\ \text{MeV c}^{-2}$. ✓

9 a i The energy of a particle in its rest frame/the energy measured when the particle is at rest relative to the observer. ✓

ii The gamma factor is $\gamma = \frac{1}{\sqrt{1 - 0.98^2}} = 5.0$. ✓
The total energy is $E = \gamma mc^2 = 5.0 \times 135 = 675\ \text{MeV}$. ✓
The momentum is $pc = \sqrt{E^2 - (mc^2)^2} = \sqrt{675^2 - 135^2} = 661\ \text{MeV}$, hence $p = 661\ \text{MeV c}^{-1}$. ✓

b i Let p_F and p_B be the momentum of the forward and backward moving photons respectively. Then the energies of these photons are $p_F c$ and $p_B c$. ✓
Conservation of energy and momentum implies $p_F - p_B = 661$ and $p_F + p_B = 675$. ✓
Adding gives $p_F = 668\ \text{MeV c}^{-1}$, i.e. an energy of 668 MeV. ✓

ii From the first equation $668 - p_B = 661$. ✓
Hence $p_B = 7.0\ \text{MeV c}^{-1}$. ✓

c i From $E = \gamma mc^2$ and $p = \gamma mv$. ✓
Eliminating the gamma factor gives the answer. ✓

ii A photon has zero rest mass and hence $E = pc$. ✓
Substituting in $v = \frac{pc^2}{E}$ gives $v = \frac{pc^2}{pc} = c$. ✓

10 a A frame of reference at rest in a gravitational field is equivalent to an accelerating frame in outer space far from all masses. ✓
If the acceleration is equal to the gravitational field strength on the planet. ✓
OR
A frame of reference that is in free fall near a massive body. ✓
Is equivalent to an inertial frame of reference in outer space far from all masses. ✓

b i Here we use the second version of the equivalence principle: consider a spacecraft in orbit around a planet and light signal sent from the back to the front of the spacecraft. ✓
The spacecraft is equivalent to an inertial from in outer space and so the light will hit exactly the front of the spacecraft. ✓
But the spacecraft is moving on a circle and so the light signal has to bend towards the planet if it is to hit the front of the spacecraft, hence the light has bent. ✓

ii Light rays follow geodesics, i.e. paths of least length. ✓
In a curved space these paths appear curved. ✓

c Light rays from the quasar bend as they go past the massive galaxy. ✓
The rays arriving at the observer on earth create multiple images when extended backwards. ✓

11 a Gravitational red-shift is the reduction of the frequency of a photon. ✓
As it climbs higher in a gravitational field. ✓

b Consider a beam of light that is emitted from the base of a box on the surface of a planet ✓
This box is equivalent to an identical box accelerating in outer space. ✓
In this box the ray of light received at the top of the box will have its frequency reduced because of the Doppler effect since the observer is moving away from the source. ✓
And therefore the same phenomenon will be observed in the box on the surface of the planet. ✓

c i $\frac{\Delta f}{f} = \frac{gH}{c^2} = \frac{9.8 \times 23}{90 \times 10^{16}} = 2.5 \times 10^{-15}$ ✓

The sign of $\frac{\Delta f}{f}$ is positive. ✓

ii The fractional change is very small. ✓
And so the emitted and received frequencies must be measured with very high precision. ✓

iii The frequency is the inverse of the period which may be thought to be the length in between the ticks of a clock. ✓
And since frequency changes the rate of ticking of the clock also changes. ✓

d In an inertial frame of reference the emitted and received frequencies will be the same. ✓
The falling elevator is equivalent to an inertial frame of reference in outer space. ✓
And so the frequency emitted and the frequency received in the elevator will also be the same. ✓

12 a i A black hole is point at which the curvature of space is infinite. The falling elevator is equivalent to an inertial frame of reference in outer space. ✓

ii The event horizon is the set of points around the black hole where the escape speed is equal to the speed of light. ✓

b $R = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 5.0 \times 10^{35}}{9.0 \times 10^{16}}$ ✓

$R = 7.4 \times 10^8$ m ✓

c The black hole constantly attracts mass from surrounding bodies into it. ✓
And as the mass increases the radius increases as well. ✓

d i From $\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{R}{r}}}$ we find $15 = \frac{5.0}{\sqrt{1 - \frac{R}{r}}}$, i.e. $1 - \frac{R}{r} = \frac{1}{9.0}$. ✓

And so $r = 1.125R = 8.3 \times 10^8$ m. ✓

ii The signal will be red-shifted. ✓